Induced representations and induced Hamiltonian actions

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Abstract. *Given a Hamiltonian action of a closed subgroup Γ of a Lie group G, we construct a Hamiltonian action of G (the induced action). Our construction follows the well known scheme of inducing unitary representations due to Mackey. In order to bring out the similarity between these constructions we reformulate Mackey's scheme in terms of «quantum reductions».*

1. INTRODUCTION

This paper is a part of a programme ([1], [2], [3]) of studying relations between the geometry of classical mechanics (i.e. symplectic geometry) and the geometry of quantum mechanics (i.e. Hilbert space geometry). It appears that many fundamental constructions working in one of those theories are performable also in the second one.

In Section 2 we present a (symplectic) concept of an induced Hamiltonian action in analogy with the known («quantum») concept of an induced unitary representation. The symplectic induction, as many other symplectic constructions, is based on the notion of a coisotropic submanifold and the corresponding symplectic reduction.

Describing the induced unitary representations (Section 3), we try to preserve the «classical» scheme as much as possible. We thus consider the construction of

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an induced representation as one more example of a «unitary reduction». Although no general theory of unitary reductions exists, a preliminary approach to such a theory was proposed in some special cases (see the works cited above).

The present work was initiated after we noticed a similarity between the construction of a coisotropic submanifold for a homogeneous representation of a Hamiltonian action, given in [4, 5], and the construction of the representation space for an induced representation in special case when $\Gamma = G$.

Throughout the paper, G denotes a finite-dimensional Lie group, Γ – its closed subgroup, $\mathcal{L}(G)$ and $\mathcal{L}(\Gamma)$ – the corresponding Lie algebras, $\mathcal{L}(G)^*$ and $\mathcal{L}(\Gamma)^*$ – their dual spaces, and $r, l : G \to \mathrm{Diff}(G)$ denote two actions of G on itself given by

$$r_g h = hg^{-1}, \quad l_g h = gh$$

for $g, h \in G$.

2. INDUCED HAMILTONIAN ACTIONS

Let $s : \Gamma \to \mathrm{aut}(P, \omega)$ be a left action of Γ on a symplectic manifold (P, ω). We assume that this action admits a Hamiltonian mapping $H : P \to \mathcal{L}(\Gamma)^*$ (see [4] for terminology).

We denote by J^r (resp. J^l) the canonical Hamiltonian mapping corresponding to the canonical lift $\hat{r}$ (resp. $\hat{l}$) $: G \to \mathrm{aut}(T^*G)$ of r (resp. l) to T^*G. For $\eta \in T_g^*G$ we have

$$J^r(\eta) = -(T_e l_g)^* \eta \quad (\text{resp. } J^l(\eta) = (T_e r_{g^{-1}})^* \eta),$$

where $(T_e l_g)^*$ (resp. $(T_e r_{g^{-1}})^*$) is the dual of the linear mapping $T_e l_g$ (resp. $T_e r_{g^{-1}}$) $: T_e G \to T_g G$, and e is the identity in G. In the sequel we shall use the restriction $\hat{r}|_\Gamma = (r|_\Gamma)^\wedge$ of $\hat{r}$ to Γ. The corresponding Hamiltonian mapping is given by $\iota^* \circ J^r$, where ι^* is the dual of the inclusion $\iota : \mathcal{L}(\Gamma) \to \mathcal{L}(G)$.

Let us consider the product action $\tilde{s} = (\hat{r}|_\Gamma, s)$ of Γ on $(\tilde{P}, \tilde{\omega})$, where $\tilde{P} = T^*G \times P$ and $\tilde{\omega}$ is naturally defined. This action admits the Hamiltonian mapping $\tilde{H} : T^*G \times P \ni (\eta, p) \to \iota^* \circ J^r(\eta) + H(p) \in \mathcal{L}(\Gamma)^*$.

Let $C = \tilde{H}^{-1}(0)$. Then

1) C is a coisotropic submanifold of $(\tilde{P}, \tilde{\omega})$ invariant under the action $\tilde{s}$, because $0 \in \mathcal{L}(\Gamma)^*$ is a regular value of $\tilde{H}$ (see [4]). C is a subbundle of the bundle $T^*G \times P$ over $G \times P$ (with the natural projection) and the fiber

$$C_{(g,p)} = \{\eta \in T_g^*G : \iota^* \circ (T_e l_g)^* \eta = H(p)\}$$

of C over (g, p) is an affine subspace of T_g^*G, parallel to

$$\ker \iota^* \circ (T_e l_g)^* = [(T_e l_g)\, \mathcal{L}(\Gamma)]^0.$$

2) The characteristics of C are orbits of the action of Γ_c – the connected component of e in Γ (see [4]).

3) The action of Γ (resp. Γ_c) on C is free and proper, since the action $r|_\Gamma$ of Γ (resp. $r|_{\Gamma_c}$) on G is free and proper. It follows that the quotient space C/Γ (resp. C/Γ_c) admits a unique differential structure such that the canonical projection is a submersion ([6]). Therefore $(\widetilde{P}, \widetilde{\omega})$ is globally reducible by C and $P_{[C]} = C/\Gamma_c$. Also $P^{\text{Ind}} = C/\Gamma$ is a symplectic manifold in a natural way (i.e. $\widetilde{\omega}|_C$ is projectible on C/Γ) and $P_{[C]}$ is a covering of P^{Ind}:

$$P^{\text{Ind}} = P_{[C]}/(\Gamma/\Gamma_c).$$

4) C is invariant under left translations by the elements of G,

$$\widetilde{l} = (\hat{l}, \text{id}) : G \to \text{aut}\,(\widetilde{P}, \widetilde{\omega})$$

$$\widetilde{l}_g(\eta, p) = (\hat{l}_g\eta, p) \qquad \text{for} \qquad \eta \in T^*G,\ p \in P,$$

since $J^r \circ \hat{l}_g = J^r$. The action $\widetilde{l}$ commutes with $\widetilde{s}$ and therefore defines a symplectic action of G on P^{Ind}. The latter action is said to be the action *induced* by s and is denoted by s^{Ind}.

It is known that a Hamiltonian mapping is constant on characteristics, therefore it can be projected on the reduced space. In our case, the Hamiltonian mapping $\widetilde{J}^l$ corresponding to $\widetilde{l}$ is constant even on orbits of $\widetilde{s}$ in $\widetilde{P}$, since

$$\widetilde{J}^l(\widetilde{s}_\gamma(\eta, p)) = J^l(\hat{r}_\gamma\eta) = J^l(\eta) = \widetilde{J}^l(\eta, p).$$

It follows that $\widetilde{J}^l$ can be projected on P^{Ind} and s^{Ind} is a Hamiltonian action.

Remarks

(i) If Γ is connected and $\Gamma = G$ then we obtain the construction of C presented in [4].

(ii) For $\Gamma = G$ we have $P^{\text{Ind}} = P$ and $s^{\text{Ind}} = s$.

3. INDUCED UNITARY REPRESENTATIONS

3.1. Preliminaries

Let $\dim G = n$, $\dim \Gamma = k$ and let δ_Γ be the modular function on Γ. Let $\pi : G \to G/\Gamma$ be the canonical projection. The transformation of G/Γ induced by the left translation l_g will be denoted by $\pi(l_g)$. For $g \in G$ we have $\pi \circ l_g = \pi(l_g) \circ \pi$.

In this section μ denotes a strictly positive density on Γ at the point $e \in \Gamma$. In other words, $\mu : \overset{k}{\wedge} \mathcal{L}(\Gamma) \to [0, \infty[$, $\mu^{-1}(0) = \{0\}$ and $\mu(tw) = |t|\,\mu(w)$ for $t \in \mathbb{R}$, $w \in \overset{k}{\wedge} \mathcal{L}(\Gamma)$. The left-invariant density on Γ corresponding to μ will be denoted by $d\mu_l$.

Let $w \in \overset{k}{\bigwedge} \mathcal{L}(\Gamma)$ be such that $\mu(w) = 1$ ($-w$ also will do; the sign is not important in the sequel). We denote by w_r the field of k-covectors on G which corresponds to w via the infinitesimal version of the action $r|_\Gamma : \Gamma \to \mathrm{Diff}(G)$. The field w_r is left-invariant and tangent to the orbits of $r|_\Gamma$, i.e. $\pi_*(w_r) = 0$.

With each density ρ on G/Γ we associate the density $\mu \otimes \rho$ on G defined by

$$(\mu \otimes \rho)_g (w_r(g) \wedge u) = \rho_{\pi(g)}(\pi_* u) \quad \text{for} \quad u \in \overset{n-k}{\bigwedge} T_g G.$$

We have $t\mu \otimes \rho = t(\mu \otimes \rho)$ for $t > 0$.

PROPOSITION 1. *The mapping $\rho \to \mu \otimes \rho$ establishes a one-to-one correspondence between densities on G/Γ and densities $\tilde{\rho}$ on G satisfying*

$$(1) \qquad r^*_{\gamma^{-1}} \tilde{\rho} = \delta_\Gamma(\gamma) \tilde{\rho} \qquad \textit{for} \qquad \gamma \in \Gamma.$$

This mapping commutes with left translations:

$$l^*_g(\mu \otimes \rho) = \mu \otimes \pi(l_g)^* \rho \qquad \textit{for} \qquad g \in G.$$

Proof. Densities of the form $\mu \otimes \rho$ satisfy (1) since

$$(r^*_{\gamma^{-1}}(\mu \otimes \rho))_g (w_r(g) \wedge u) = (\mu \otimes \rho)_{g\gamma}(r^{-1}_{\gamma *} w_r \wedge r^{-1}_{\gamma *} u) =$$
$$= (\mu \otimes \rho)_{g\gamma}(\det(Ad_{\gamma^{-1}}) \cdot w_r \wedge r^{-1}_{\gamma *} u) = |\det Ad_{\gamma^{-1}}|\, (\mu \otimes \rho)_{g\gamma}(w_r \wedge r^{-1}_{\gamma *} u) =$$
$$= \delta_\Gamma(\gamma) \rho_{\pi(g)\gamma}(\pi_* r^{-1}_{\gamma *} u) = \delta_\Gamma(\gamma) \rho_{\pi(g)}(\pi_* u) = \delta_\Gamma(\gamma)(\mu \otimes \rho)_g (w_r(g) \wedge u).$$

Conversely, if $\tilde{\rho}$ satisfies (1) then the equality

$$\rho_{\pi(g)}(\pi_* u) = \tilde{\rho}_g(w_r(g) \wedge u)$$

defines correctly a density ρ on G/Γ such that $\tilde{\rho} = \mu \otimes \rho$. The last assertion of the proposition follows from the left-invariance of w_r. ∎

We denote by $\mathcal{D}(G)$ the space of smooth half-densities with compact supports on G and by $L^2(G)$ the completion of $\mathcal{D}(G)$ with respect to the natural scalar product $\langle \phi | \psi \rangle_G = \int_G \bar{\phi} \cdot \psi$.

3.2. Induced representations: the problem of reduction

Let $U : \Gamma \to \mathrm{aut}(V, \langle \cdot | \cdot \rangle_V)$ be a unitary representation of Γ in a Hilbert space V.

Let R (resp. L) : $G \to \mathrm{aut}\, L^2(G)$ denote the canonical lift of r (resp. l) to $L^2(G)$, i.e.

$$R_g \psi = r^*_{g^{-1}} \psi \quad (\text{resp. } L_g \psi = l^*_{g^{-1}} \psi) \quad \text{for } g \in G,\ \psi \in L^2(G).$$

Let us consider the product representation $\gamma \to \widetilde{U}_\gamma = R_\gamma \otimes U_\gamma$ of Γ in $\widetilde{V} = L^2(G) \underline{\otimes} V$ ($\underline{\otimes}$ denotes the usual tensor product of Hilbert spaces). Roughly speaking, the induced representation is the restriction of the representation $g \to L_g \otimes \mathrm{id}$ to the subspace of vectors which are invariant under $\widetilde{U}$ (such a subspace would be invariant for $L_g \otimes \mathrm{id}$ since L commutes with R). The «only» problem is that in general there are no vectors invariant under $\widetilde{U}$ in $\widetilde{V}$ (unless Γ is compact).

The above problem is typical for a quantum analogue of the classical symplectic reduction. In our previous papers we used «Gelfand triples» to solve such problems. We observed that «reduced Hilbert spaces» can be constructed from subspaces of generalized vectors. The representation space of the induced representation has to be constructed like in the symplectic case – via reduction. Elements of this Hilbert space will be represented by «generalized vectors in $\widetilde{V}$» invariant under $\widetilde{U}$.

3.3. Gelfand triple

$\widetilde{U}$ acts naturally on spaces larger than $\widetilde{V}$. An example of such a space can be constructed as follows. Let $\mathscr{D}(G, V)$ denote the space of smooth half-densities on G with compact supports and with values in V. We have $\mathscr{D}(G, V) = \mathscr{D}(G) \bar{\otimes} V$ (see [7]), $\mathscr{D}(G)$ and $\mathscr{D}(G, V)$ being equipped with their usual topologies. $\mathscr{D}(G, V)$ is a dense subspace of $\widetilde{V}$ (the elements of $\widetilde{V}$ may be represented as (classes of measurable) square-integrable half-densities on G with values in V). The scalar product of $\phi, \psi \in \mathscr{D}(G, V)$ is given by

$$\langle \phi \mid \psi \rangle_{\widetilde{V}} = \int_G \langle \phi \mid \psi \rangle_V .$$

Let $\mathscr{D}(G, V)^x$ denote the space of continuous anti-linear functionals on $\mathscr{D}(G, V)$ ($\mathscr{D}(G, V)$ can be identified with the space of continuous linear mapping from $\mathscr{D}(G)$ to V, see [7]). The value of $T \in \mathscr{D}(G, V)^x$ on $\psi \in \mathscr{D}(G, V)$ will be denoted by $< \psi \mid T)$. Now $\widetilde{V}$ can be identified with a linear subspace of $\mathscr{D}(G, V)^x$. The inclusion $\alpha : \widetilde{V} \to \mathscr{D}(G, V)^x$ is given by

$$\langle \psi \mid \alpha(h)) = \langle \psi \mid h \rangle_{\widetilde{V}} \qquad \text{for} \qquad h \in \widetilde{V},\ \psi \in \mathscr{D}(G, V).$$

For each $\gamma \in \Gamma$, $\widetilde{U}_\gamma$ maps $\mathscr{D}(G, V)$ on itself. We denote the inverse of the corresponding adjoint transformation of $\mathscr{D}(G, V)^x$ by the same letter because it extends the original $\widetilde{U}_\gamma$. More exactly, we put

$$\langle \psi \mid \widetilde{U}_\gamma T) = \langle \widetilde{U}_{\gamma^{-1}} \psi \mid T) \qquad \text{for} \quad T \in \mathscr{D}(G, V)^x,\ \psi \in \mathscr{D}(G, V).$$

In the sequel we shall use the Gelfand triple

$$\mathscr{D}(G, V) \subset \widetilde{V} \subset \mathscr{D}(G, V)^x$$

in our construction of the reduced space.

3.4. The reduced space as a subspace of $\mathscr{D}$(G, V)^x

The idea is to choose a subspace of sufficiently regular elements of $\mathscr{D}(G, V)^x$ which are invariant under $\widetilde{U}$ and try to define a G-invariant scalar product for them. To be specific, let us consider the space $\xi(G, V)$ of smooth half-densities on G with values in V. Proposition 1 suggests to consider not strictly invariant elements of $\xi(G, V)$ but rather elements satisfying

(2) $$\widetilde{U}_\gamma \psi = \delta_\Gamma(\gamma)^{1/2} \psi \quad \text{for} \quad \gamma \in \Gamma.$$

For such an element ψ, $\langle \psi \mid \psi \rangle_V$ satisfies (1). Let ξ_0^{Ind} be the subspace of $\xi(G, V)$ composed of elements ψ satisfying (2) and such that $\langle \psi \mid \psi \rangle_V/\mu$ has a compact support. Here $\tilde{\rho} \to \tilde{\rho}/\mu$ denotes the inverse of the mapping $\rho \to \mu \otimes \rho$. For ϕ, $\psi \in \xi_0^{\mathrm{Ind}}$ we put

$$\langle \phi \mid \psi \rangle_\mu = \int_{G/\Gamma} \langle \phi \mid \psi \rangle_V/\mu.$$

This is a G-invariant scalar product:

$$\langle L_g\phi \mid L_g\psi \rangle_\mu = \int_{G/\Gamma} \langle L_g\phi \mid L_g\psi \rangle_V/\mu = \int_{G/\Gamma} l^*_{g^{-1}}\langle \phi \mid \psi \rangle_V/\mu =$$

$$= \int_{G/\Gamma} \pi(l_{g^{-1}})^*(\langle \phi \mid \psi \rangle_V/\mu) = \int_{G/\Gamma} \langle \phi \mid \psi \rangle_V/\mu = \langle \phi \mid \psi \rangle_\mu .$$

We denote by V^{Ind} the completion of ξ_0^{Ind} with respect to $\langle \, . \mid . \, \rangle_\mu$. The representation $g \to L_g$ of G in ξ_0^{Ind} gives rise to a representation of G in V^{Ind}, called the induced representation.

3.5. The projection operator associated with the reduction

In the case of compact Γ we have the projection on the subspace of invariant vectors, given by

(3) $$p_\mu = \int_\Gamma \widetilde{U}_\gamma \, \mathrm{d}\mu_l(\gamma)$$

(in this case μ is fixed by the condition $\int_\Gamma \mathrm{d}\mu_l(\gamma) = 1$).

A formula of this kind can be used also in a general case, provided we apply it to sufficiently regular vectors. The range of p_μ is then composed of certain generalized vectors. In the case when Γ is not unimodular we must modify (3) in order to get (at least formally) a self-adjoint expression. The right formula is the following:

$$p_\mu = \int_\Gamma \delta_\Gamma(\gamma)^{-1/2} \widetilde{U}_\gamma \, \mathrm{d}\mu_l(\gamma).$$

Indeed, formally we have

$$p_\mu^* = \int_\Gamma \delta_\Gamma(\gamma)^{-1/2} \widetilde{U}_\gamma^* \mathrm{d}\mu_l(\gamma) = \int_\Gamma \delta_\Gamma(\gamma^{-1})^{-1/2} \widetilde{U}_\gamma \, \mathrm{d}\mu_l(\gamma^{-1}) =$$

$$= \int_\Gamma \delta_\Gamma(\gamma)^{1/2} \widetilde{U}_\gamma \delta_\Gamma(\gamma)^{-1} \mathrm{d}\mu_l(\gamma) = p_\mu.$$

Vectors from the range of p_μ satisfy (2).

Now we specify the domain of p_μ. It is easy to prove that for $\psi \in \mathscr{D}(G, V)$ the integral

$$p_\mu \psi = \int_\Gamma \delta_\Gamma(\gamma)^{-1/2} \widetilde{U}_\gamma \psi \, \mathrm{d}\mu_l(\gamma) \tag{4}$$

defines pointwise a smooth half-density on G with values in V. Therefore $p_\mu : \mathscr{D}(G, V) \to \xi_0^{\mathrm{Ind}} \subset \mathscr{D}(G, V)^x$. It was shown in [7] that $p_\mu(\mathscr{D}(G, V)) = \xi_0^{\mathrm{Ind}}$. Let us notice the following properties of p_μ:

$$\widetilde{U}_\gamma p_\mu = \delta_\Gamma(\gamma) p_\mu \widetilde{U}_\gamma = \delta_\Gamma(\gamma)^{1/2} p_\mu \quad \text{for} \quad \gamma \in \Gamma$$

and

$$L_g p_\mu = p_\mu L_g \quad \text{for} \quad g \in G. \tag{5}$$

PROPOSITION 2. *If* $\phi, \psi \in \mathscr{D}(G, V)$ *then* $\langle \phi \mid p_\mu \psi \rangle = \langle p_\mu \phi \mid p_\mu \psi \rangle_\mu$.

Proof. We use the following version of the Fubini theorem:

$$\int_G \rho = \int_{G/\Gamma} p_\mu^1 \rho / \mu,$$

where ρ is a smooth density with compact support on G and

$$p_\mu^1 \rho = \int_\Gamma \delta_\Gamma(\gamma)^{-1} r^*_{\gamma^{-1}} \rho \, \mathrm{d}\mu_l(\gamma)$$

($p_\mu^1 \rho$ satisfies (1)). We obtain

$$\langle \phi \,|\, p_\mu \psi \rangle = \int_G \langle \phi \,|\, p_\mu \psi \rangle_V = \int_{G/\Gamma} p_\mu^1 \langle \phi \,|\, p_\mu \psi \rangle_V / \mu .$$

But

$$p_\mu^1 \langle \phi \,|\, p_\mu \psi \rangle_V = \int_\Gamma \delta_\Gamma(\gamma)^{-1} r^*_{\gamma^{-1}} \langle \phi \,|\, p_\mu \psi \rangle_V \, \mathrm{d}\mu_l(\gamma)$$

and

$$\begin{aligned} r^*_{\gamma^{-1}} \langle \phi \,|\, p_\mu \psi \rangle_V &= \langle R_\gamma \phi \,|\, R_\gamma p_\mu \psi \rangle_V = \\ &= \langle R_\gamma \phi \,|\, U_{\gamma^{-1}} \delta_\Gamma(\gamma)^{1/2} p_\mu \psi \rangle_V = \delta_\Gamma(\gamma)^{1/2} \langle \widetilde{U}_\gamma \phi \,|\, p_\mu \psi \rangle_V , \end{aligned}$$

hence

$$p_\mu^1 \langle \phi \,|\, p_\mu \psi \rangle_V = \int_\Gamma \delta_\Gamma(\gamma)^{-1/2} \langle \widetilde{U}_\gamma \phi \,|\, p_\mu \psi \rangle_V \mathrm{d}\mu_l(\gamma) = \langle p_\mu \phi \,|\, p_\mu \psi \rangle_V . \qquad \blacksquare$$

COROLLARY. *$\langle \cdot \,|\, p_\mu \cdot \rangle$ is a positive continuous Hermitian form on $\mathscr{D}(G, V)$.* ■

3.6. The reduced space as completion of a quotient space of $\mathscr{D}$(G, V)

One can construct a Hilbert space using the Hermitian positive form $\langle \cdot \,|\, p_\mu \cdot \rangle$ on $\mathscr{D}(G, V)$ in a standard way, as the completion of the quotient space $\mathscr{D}(G, V)/\ker p_\mu$ with respect to the induced (non-degenerate) Hermitian form. The projection operator p_μ defines a bijection between the quotient space and ξ_0^{Ind}. Proposition 2 shows that this bijection preserves the scalar product. By formula (5) ker p_μ and the form $\langle \cdot \,|\, p_\mu \cdot \rangle$ are invariant under L_g. Hence L_g can be projected on the quotient space. This gives an alternative (equivalent) description of the induced representation.

3.7. Concluding remarks

We summarize this section as follows. In course of making the analogy between the classical and the quantum case more explicit, we have found the correct condition for the functions belonging to the representation space (i.e. equation (2)). We intended to find a simple key to a better understanding of this condition. We have obtained the condition in two different ways:

(i) asking to what objects on G there correspond the densities on G/Γ (Prop. 1).

(ii) looking for a positive (in particular self-adjoint) operator of projection on (almost) invariant vectors.

This projection is given by formula (4) which can be written also in the following form

$$p_\mu = \int_\Gamma \tilde{U}_\gamma \, d\mu \qquad d\mu = (d\mu_r \, d\mu_l)^{1/2},$$

where $d\mu_r(\gamma) = d\mu_l(\gamma^{-1})$ is the right-invariant measure on Γ. The measure $d\mu$ is invariant under reflection $\gamma \mapsto \gamma^{-1}$.

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